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Interface properties of the two-dimensional Blume–Emery–Griffiths model

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Abstract. We study the interface behaviour of the two-dimensional Blume–Emery–Griffiths model which is described by the Hamiltonian

$$H = -J \sum_{\langle ij \rangle} S_i S_j - K \sum_{\langle ij \rangle} S_i^2 S_j^2 + D \sum_i S_i^2 \quad S_i = 1, 0, -1.$$

An interface is introduced into the system by fixing the spins on opposite boundaries in two different states, +1 and -1. We pay particular attention to the appearance of an excess of the third state, 0, in the vicinity of the interface. The net adsorption of the non-boundary state is studied near critical, first-order and tricritical transitions.

Two methods have been used to attack this problem, the Monte Carlo technique and a modified version of the interface free energy approximation of Müller-Hartmann and Zittartz which is seen to give a surprisingly good description of interface properties in three-state systems.

1. Introduction

Interfaces between two phases (α , γ) in the presence of a third (β) phase have been studied both experimentally and theoretically. In particular, it has been found that at the, say, $\alpha\gamma$ interface a β layer may be formed. This effect has been studied experimentally, for example, by Moldover and Cahn (1980), and described within a classical, phenomenological framework by Widom (1978). Related theoretical studies have also been given by e.g. de Gennes (1981) and Abraham and Smith (1982). However, it has only been appreciated recently that a similar effect occurs at interfaces in microscopic three-state models. In this case an interface is introduced by fixing the variables at opposite boundaries in two different states. Using Monte Carlo (MC) techniques it has been observed, for the two-dimensional three-state Potts model, that an excess of the third, non-boundary state is generated at the interface mostly in the form of droplets (Selke and Pesch 1982). The critical behaviour of the net adsorption of non-boundary states at the interface has been studied and the singular behaviour has been found to be governed by the bulk correlation length in agreement with a simple scaling argument (Selke and Pesch 1982, Selke and Huse 1983). These results may be of experimental relevance for certain adsorbed monolayer systems (for a review see Schick 1981). One should caution, however, that Potts models violate Antonoff's rule (Widom 1975) in contrast to multicomponent fluids.

Multicomponent fluids may be modelled in a more realistic manner (Lajzerowicz and Sivardiere 1975, Mukamel and Blume 1974) by the Blume–Emery–Griffiths (BEG) model (Blume *et al* 1971) or its special case, the Blume–Capel (BC) model (Capel 1966, Blume 1966). Introducing an interface in these spin-1 models one may expect, by analogy with the Potts case, the formation of an intermediate layer of non-boundary states. Using standard MC techniques (Binder 1979) for the two-dimensional BC model we indeed observe this phenomenon. In particular, we study the critical behaviour of the interfacial adsorption of non-boundary states associated with different classes of bulk phase transitions (second-order, tricritical, first-order). Results are presented in § 2.

The occurrence of the intermediate layer at the interface provides the clue for the modification of the method of Müller-Hartmann and Zittartz (1977) (originally devised to map phase diagrams of two-component systems) to treat the two-dimensional BEG model; the analogous method for the three-state Potts model is presented in Selke and Pesch (1982). In § 3 we discuss the extent to which this method is able to describe the phase diagram of the BEG model. The results, obtained with modest numerical effort, are of an accuracy comparable to those predicted by rather involved real-space renormalisation-group calculations (Berker and Wortis 1976, Burkhardt 1976). Section 4 provides a brief summary.

2. Monte Carlo results

The two-dimensional BEG model (Blume *et al* 1971) is described by the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - K \sum_{\langle ij \rangle} S_i^2 S_j^2 + D \sum_i S_i \quad S_i = 1, 0, -1 \quad (2.1)$$

where $\langle ij \rangle$ indicates summation over nearest neighbours on a square lattice of size $N \times N$. The terms on the right-hand side represent the usual bilinear exchange ($J > 0$), biquadratic exchange ($K > 0$) and crystal-field ($D > 0$) interactions.

The Monte Carlo results described here are restricted to the case $K/J = 0$. This corresponds to the BC model (Capel 1966, Blume 1966), which was originally suggested to describe an Ising system of spin-1 ions subject to a zero-field splitting. The model is known to display a second-order transition for $0 \leq D/J < (D/J)_t$, tricritical point at $(D/J)_t$ (using MC renormalisation-group techniques $(D/J)_t$ has been determined very precisely as $D_t \approx 1.931$ (Landau and Swendsen 1981, 1983)) and a first-order transition in the range $(D/J)_t < D/J \leq 2$.

Our MC results confirm this picture (see figure 11 in § 3). The critical temperature, T_c , of the second-order line was obtained from a finite size analysis ($6 \leq N \leq 40$) of the location of the maximum in the specific heat. The typical length of a run for a given set of parameters ($D/J, k_B T/J$) was between 5×10^3 and 10^4 Monte Carlo steps per site (MCS/s). Our estimates of T_c ($N = \infty$) agree very well with previous MC results (Arora and Landau 1971, Jain 1976). Accurate determination of T_c at the first-order transitions is considerably more difficult. We compared three different methods: (i) 'rule of equal areas' of the hysteresis loop; (ii) matching of the free energy of the low-temperature and high-temperature branch (Binder 1982, Liebmann 1982); (iii) disentanglement of MC configurations with half of the spins in the high-temperature phase, $S_i = 0$, and the other half in the low-temperature phase, for example $S_i = 1$

(Creutz *et al* 1979). In this way we obtained consistent and apparently reliable estimates with small error bars.

To introduce an interface into the BC model the spins on opposite boundaries were fixed in the two different non-zero states, ± 1 . At zero temperature, for $D/J < 1$, the interface is a straight line separating the 'up' and 'down' domains. It is interesting to note that for $1 < D/J$, however, the ground-state energy is lowered by the insertion of a $S_i = 0$ monolayer at the interface. By examining typical equilibrium MC configurations at non-zero temperatures, as shown in figure 1, it is seen that an excess of the non-boundary state, $S_i = 0$, is included at the interface. The interface's adsorption of zeros occurs in a layer-like fashion as expected on the basis of single spin-flip energy considerations. This should be compared to the droplet-like adsorption of non-boundary states in Potts models (Selke and Pesch 1982) which occurs because all states are energetically equivalent.

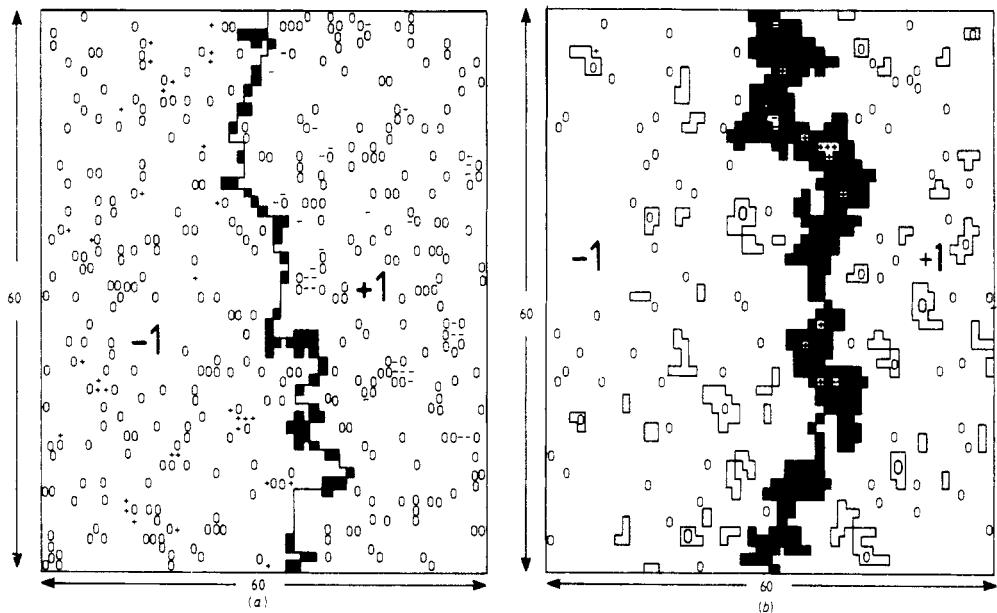


Figure 1. Typical equilibrium Monte Carlo configurations showing the excess of the non-boundary state, $S_i = 0$, (shown in black) at the interface. The temperatures and fields are chosen so that the system is close to (a) the critical temperature and (b) the tricritical point. Note that in (b) the width of the interface layer is increased because the system is close to three-phase coexistence. In (a) the value of $D/J = 0$ and $k_B T/J = 1.52$ and in (b) $D/J = 1.92$ and $k_B T/J = 0.64$.

The thickness of the insertion layer at the interface depends on the value of D/J , the size of the system, N , and the temperature. To describe the phenomenon quantitatively we define the net adsorption per unit length of the interface, by analogy to the Potts case, as

$$W_0 = \frac{1}{N} \sum_i (\langle \delta_{0,S_i} \rangle_{1:-1} - \langle \delta_{0,S_i} \rangle_{1:1}) \quad (2.2)$$

where the angle brackets denote thermal averages and the subscripts $1:-1$ and $1:1$

refer to systems with and without interfaces, respectively. W_0 may be also considered as a measure of the thickness of the idealised adsorption layer at the interface, which contains *only* spins $S_i = 0$. The MC results were obtained by studying systems of sizes of $N \times N$ with $6 \leq N \leq 100$. All data points are averages over runs of at least 5×10^3 MCS/s; in case of large thermal fluctuations close to T_c , several runs were done for a given set of parameters ($D/J, k_B T/J, N$).

The non-boundary states generated at the interface involve an energy cost which is compensated by a gain in entropy. This compensation is clearly seen in the temperature dependence of W_0 . The behaviour of W_0 as a function of temperature and the size of the system, N , is shown in figures 2 and 3. W_0 displays a maximum near T_c the height of which increases with increasing D/J . This is easily understood by recalling that the crystal field interaction favours spins in the state $S_i = 0$. The height of the peak also increases with increasing N and the maximum occurs at a value of the temperature which approaches T_c , albeit rather slowly, with increasing N . To investigate this point further in figure 4 the size dependence is shown for $D/J = 0$ where the critical temperature is known accurately from high-temperature series expansions to be $k_B T_c/J \approx 1.69$ (Fox and Guttmann 1973, Burkhardt and Swendsen 1976). Figure 4 strongly suggests that the maximum of W_0 occurs at T_c in the thermodynamic limit.

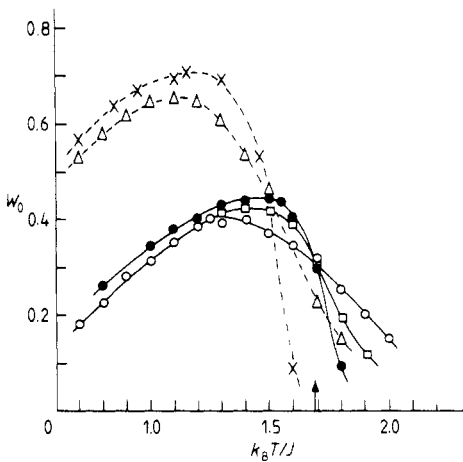


Figure 2. Net adsorption, W_0 , of the non-boundary state of the interface as a function of temperature, $k_B T/J$, and the system size, N . The values of N are: \times , 40; Δ , 20; \bullet , 40; \circ , 10; \square , 20. $D/J = 0.8$ for the broken curves and $D/J = 0$ for the full curves.

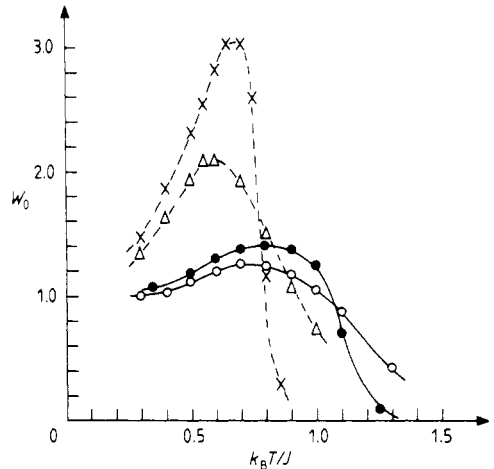


Figure 3. Net adsorption, W_0 , of the non-boundary state at the interface as a function of temperature, $k_B T/J$, and the system size, N . The values of N are: \times , 40; Δ , 10; \bullet , 40; \circ , 10. $D/J = 1.9$ for the broken curves and $D/J = 1.6$ for the full curves.

Note also, that for $T > T_c$, W_0 approaches zero rather rapidly, particularly for large values of N . This is consistent with the vanishing of the interface above T_c ; indeed, we expect that in the thermodynamic limit the interface has zero width for all $T > T_c$. Note that we may determine T_c with modest numerical effort quite reliably from the point of intersection of the curves with different N (see figures 2 and 3).

The most interesting behaviour of W_0 occurs near T_c . We therefore studied both the temperature and the size dependence of the net adsorption W_0 in the critical

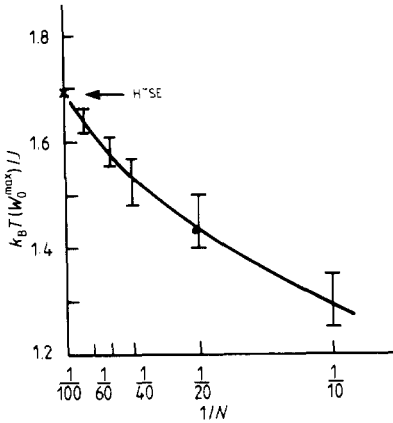


Figure 4. Size dependence of the height of the maximum in the net adsorption, W_0^{\max} , for $D/J=0$. The estimated value of W_0^{\max} for $N=\infty$ agrees well with the high-temperature series expansion result (HTSE), thus providing evidence that the maximum of W_0 occurs at the critical temperature in the thermodynamic limit.

region. To analyse the size dependence (and establish a possible divergence of W_0 at T_c) we determined the height of W_0 , W_0^{\max} , for chosen values of D/J as a function of N . Results are shown in figures 5 and 6 for $D/J \leq (D/J)_i \approx 1.931$ (Landau and Swendsen 1981, 1983). For $D/J = 0, 0.4, 0.8, 1.2, 1.6$ and 1.8 $W_0^{\max}(N)$ can be

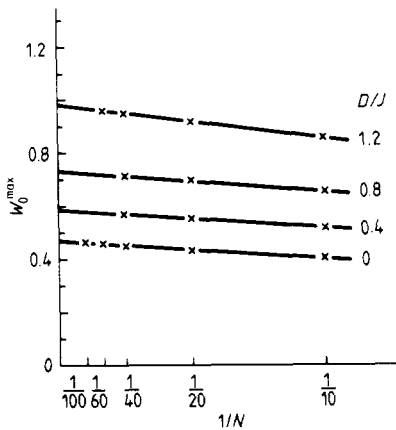


Figure 5. Size dependence of the maximum value of the net adsorption, W_0^{\max} , for chosen values of the crystal field, D/J .

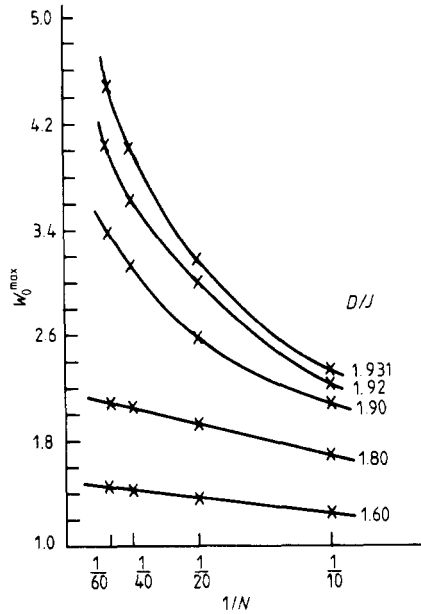


Figure 6. Size dependence of the maximum value of the net adsorption, W_0^{\max} , for chosen values of the crystal field, D/J . Note the change in the behaviour of the curves as the tricritical point is approached.

expressed in a simple form

$$W_0^{\max}(N) = W_0^{\max}(\infty) - C(D/J)/N \quad 10 \leq N \leq 60 \quad (2.3)$$

where $W_0^{\max}(\infty)$ is finite and both $W_0^{\max}(\infty)$ and the coefficient C increase monotonically with D/J . Corrections to (2.3) appear very small. Extrapolating (2.3) to the thermodynamic limit indicates that the net adsorption does not diverge at the second-order transition of the two-dimensional Blume–Capel model. This is in contrast to the Potts case (Selke and Huse 1983). However, the size dependence of W_0^{\max} changes drastically as the tricritical point is approached. Results for $D/J = 1.9, 1.92$ and 1.931 are shown in figure 6. For $10 \leq N \leq 60$ the data can be fitted by

$$W_0^{\max}(N) \sim \ln N \text{ or } N^a \text{ with } a \leq 0.2 \quad (2.4)$$

as demonstrated in figure 7. Note that the slope, $d(W_0^{\max})/d(\ln N)$, decreases appreciably with increasing N giving an upper bound for a : similarly for ω (see below). It seems likely that the qualitative change in the size dependence can be attributed to a crossover effect resulting from the closeness of the tricritical point. Additional evidence for a divergent net adsorption at the bulk value of T_c may be obtained by analysing the critical temperature dependence. At $D/J = 1.92$ (for fixed $N = 60$) we find a logarithmic divergence (or a power-law behaviour with a small exponent) in W_0 as a function of the reduced temperature $t = (T_c - T)/T_c$:

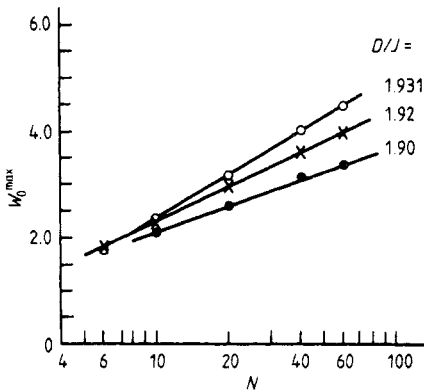


Figure 7. To show that the maximum value of the net adsorption, W_0^{\max} , varies logarithmically with the size of the system, N , for three values of the crystal field, D/J , which lie close to the tricritical point.

$$W_0(t) \sim \ln t \text{ or } t^{-\omega} \quad \text{with } \omega \leq 0.1 \quad (2.5)$$

in the range $2 \times 10^{-2} \leq t \leq 2 \times 10^{-1}$. These results are displayed in figure 8. Again, one may interpret this divergence as due to the closeness of the tricritical point, as confirmed by data at $D/J = 1.931$ ($N = 40$), where $W_0 \sim \ln t$ or $t^{-\omega}$ with $\omega < 0.15$.

Although such different behaviour is seen near the tricritical and critical transitions we must caution, as with all MC studies, that it may be possible that results have not been obtained close enough to T_c or for sufficiently large N to display the true asymptotic behaviour.

The analysis of the critical behaviour of W_0 at the first-order transitions, $(D/J)_t < D/J \leq 2$, turns out to be very difficult, mainly due to the bulk hysteresis. It will be

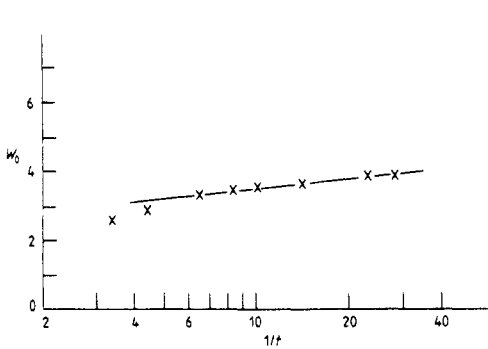


Figure 8. To show that the net adsorption, W_0 , varies logarithmically with the reduced temperature, $t = (T_c - T)/T_c$, for a value of the crystal field, $D/J = 1.92$, which lies close to the tricritical point ($N = 60$).

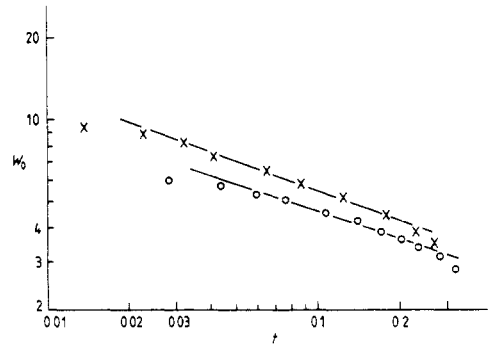


Figure 9. To show that the net adsorption, W_0 , varies as $t^{-\omega}$, where $t = (T_c - T)/T_c$, and $\omega = 0.33 \pm 0.03$ for two values of the crystal field, D/J , for which the transition is first order. The values of D/J are \circ , 1.96 and \times , 1.98. N is 40.

given elsewhere (Selke 1983). Here we summarise the two main conclusions based on data for $D/J = 1.96$ and 1.98. Choosing $N = 40$ for both values of D/J

$$W_0(t) \sim t^{-\omega} \quad \text{with } \omega = 0.33 \pm 0.03 \tag{2.6}$$

for the decade $2 \times 10^{-2} \leq t \leq 2 \times 10^{-1}$. The results on which this conclusion is based are shown in figure 9. We also find

$$W_0(T_c, N) \sim N^a \quad \text{with } a = 0.7 \pm 0.05. \tag{2.7}$$

This result is based on data for $6 \leq N \leq 80$ for both $D/J = 1.96$ and 1.98. A qualitative argument can be given which yields $\omega = \frac{1}{3}$ (Huse 1982) and an exactly soluble model, similar to that described by Abraham and Smith (1982), gives $\omega = \frac{1}{3}$ (Abraham 1982).

3. Analytic results

In this section, the two-dimensional BEG model described by the Hamiltonian (2.1) is studied, using a modified version of the Müller-Hartmann-Zittartz (MHZ) approximation (1977) in which a transition is identified by the vanishing of the interface free energy. Our aim is to investigate the accuracy and applicability of the method throughout the phase space (J, K, D) .

The interface is introduced into the system by fixing the spins on opposite boundaries to take the value $S_i = +1$ and $S_i = -1$ respectively. To obtain good estimates for the phase boundaries those configurations where a layer of the third state $S_i = 0$ is allowed to intervene at the interface must be taken into account in the calculation of the free energy (Selke and Pesch 1982). The calculation is rendered tractable by neglecting both overhangs at the interface and islands of boundary states inside the intervening layer.

The notation we shall use to describe the interface is shown in figure 10. Two integers are needed to specify the position of the interface in each column. $-\infty \leq m_i \leq \infty$ denotes the position of the boundary of the domain, $S_i = +1$, relative to a reference line in the middle of the system and $0 \leq n_i \leq \infty$ the width of the intervening layer,

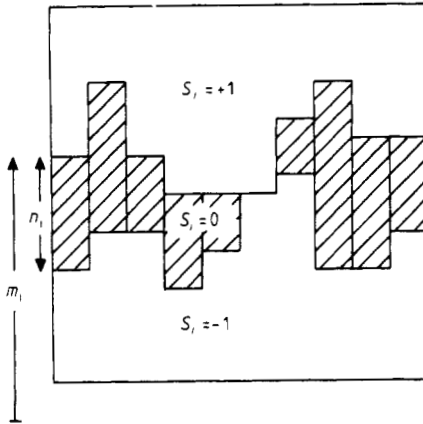


Figure 10. Schematic diagram of the interface region defining the notation used in the Müller-Hartmann-Zittartz approximation.

$S_i = 0$. Then, defining $\Delta m_i = m_{i+1} - m_i$, the total energy of a given interface configuration is

$$E = \sum_{i=1}^N [\theta(n_i - n_{i+1})E_{>}(n_i, n_{i+1}, \Delta m_i) + \theta(n_{i+1} - n_i)E_{<}(n_i, n_{i+1}, \Delta m_i) + J(n_i + 1 + \delta_{n_i,0}) + K(n_i + 1 - \delta_{n_i,0}) - D_{n_i}] \tag{3.1}$$

where

$$\begin{aligned} E_{>}(n_i, n_{i+1}, \Delta m) &= J(-2\Delta m - n_i + n_{i+1}) + K(n_i + n_{i+1}) & \Delta m < -n_i \\ & J(-\Delta m + n_{i+1}) + K(-\Delta m + n_{i+1}) & -n_i \leq \Delta m < -(n_i - n_{i+1}) \\ & Jn_i + Kn_i & -(n_i - n_{i+1}) \leq \Delta m < 0 \\ & J(\Delta m + n_i) + K(\Delta m + n_i) & 0 \leq \Delta m < n_{i+1} \\ & J(2\Delta m + n_i - n_{i+1}) + K(n_i + n_{i+1}) & n_{i+1} \leq \Delta m \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} E_{<}(n_i, n_{i+1}, \Delta m) &= J(-2\Delta m - n_i + n_{i+1}) + K(n_i + n_{i+1}) & \Delta m \leq n_i \\ & J(-\Delta m + n_{i+1}) + K(-\Delta m + n_{i+1}) & -n_i \leq \Delta m < 0 \\ & Jn_{i+1} + Kn_{i+1} & 0 \leq \Delta m < n_{i+1} - n_i \\ & J(\Delta m + n_i) + K(\Delta m + n_i) & n_{i+1} - n_i \leq \Delta m < n_{i+1} \\ & J(2\Delta m + n_i - n_{i+1}) + K(n_i + n_{i+1}) & n_{i+1} \leq \Delta m. \end{aligned} \tag{3.3}$$

The free energy of the interface is then defined in the usual way by

$$F = -k_B T \lim_{N \rightarrow \infty} \frac{1}{N} \ln \sum_{\{m_i, n_i\}} \exp(-E/k_B T). \tag{3.4}$$

The summation over m_i and m_{i+1} can be performed easily, because the sum is a function only of the difference $\Delta m = m_{i+1} - m_i$. The resulting expression for E depends only on n_i and n_{i+1} and hence the configuration sum in (3.4) can be replaced by a

product of transfer matrices, T ,

$$F = -k_B T \lim_{N \rightarrow \infty} \frac{1}{N} \ln \mathbf{T}^N \tag{3.5}$$

where

$$\mathbf{T}_{ij} = \exp(-J(j+1) - K(j+1) - \delta_{i,0}(J-K) + Dj) \begin{cases} \alpha^+ & i \geq j \\ \alpha^- & i < j \end{cases} \tag{3.6}$$

$$\alpha^+ = e^{-(J+K)(i+j)} [(1 + e^{-2J}) / (1 - e^{-2J})] + e^{-(J+K)i} [(1 + e^{-(J+K)}) (1 - e^{-(J+K)j}) / (1 - e^{-(J+K)}) + (i - j)] \tag{3.8}$$

$$\alpha^- = e^{-(J+K)(i+j)} [(1 + e^{-2J}) / (1 - e^{-2J})] + e^{-(J+K)j} [(1 + e^{-(J+K)}) (1 - e^{-(J+K)i}) / (1 - e^{-(J+K)}) + (j - i)]. \tag{3.9}$$

In the thermodynamic limit the largest eigenvalue of \mathbf{T} , λ_0 , dominates the product in (3.5) and this equation simplifies to

$$F = -k_B T \ln \lambda_0. \tag{3.10}$$

Therefore, the interface free energy is easily obtained from a numerical calculation of the largest eigenvalue of \mathbf{T} . Away from the first-order phase boundary only a small number of elements of \mathbf{T} need be considered to gain convergence of the eigenvalue.

(i) $K/J = 0$

For $K/J = 0$ the BEG model reduces to the BC model. As described in § 2 the phase diagram exhibits a first-order transition line (which may, in fact, be identified as a triple line where *three* phases co-exist by studying the model in a field (Kaufman *et al* 1981)) and a critical line which join at the tricritical point. In figure 11 the results of the modified MHZ method are shown, together with results from a position space renormalisation group (Berker and Wortis 1976) and from the MC study.

Also indicated are the MC renormalisation-group determination of the position of the tricritical point at $(D/J)_t \approx 1.931$ (Landau and Swendsen 1981, 1983) and the estimate for the transition temperature of the spin-1 Ising model ($D/J = 0$) based on a high-temperature series expansion (Fox and Guttmann 1973, Burkhardt and Swendsen 1976). To study the effect on the approximation of allowing a layer of non-boundary states at the interface results were also obtained for systems in which the width of the interfacial layer was restricted to less than n_0 spins in each column.

The phase boundary obtained for $n_0 = 2$ is shown in figure 11. For large enough values of n_0 the phase diagram is in good agreement with previous results. In particular, the rapid change of direction of the phase boundary in the vicinity of the tricritical point is reproduced and the approximation is seen to be capable of predicting a line of three-phase coexistence.

For small n_0 , however, the triple line is not observed; this is to be expected as we are suppressing three-phase coexistence by restricting the width of the boundary layer of zeros at the interface. Indeed, an analytic calculation of the value of D/J at the first-order transition at zero temperature gives the exact result only if the interface layer is allowed to be infinitely wide ($n_0 = \infty$).

It is of particular interest to look at the behaviour of the free energy near the triple line. Results for $k_B T/J = 0.5$ where the transition is strongly first order are shown in figure 12 for $n_0 = 2, 10$ and 40 . Although the free energy remains analytic

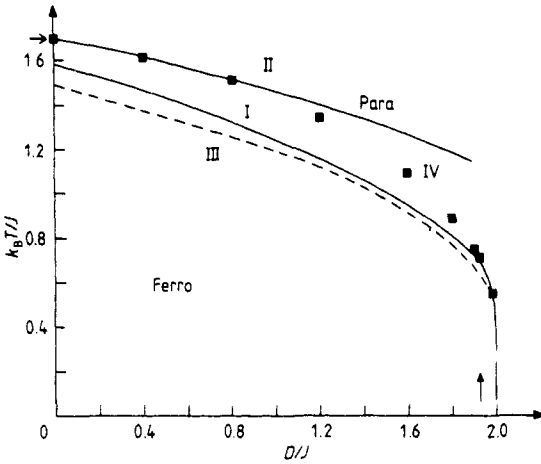


Figure 11. Phase diagram of the BEG model for $K/J = 0$ calculated using the modified Müller-Hartmann and Zittartz approximation (I). The results are compared to those obtained using the same approximation but allowing no more than one spin of value $S_i = 0$ to interpolate at the interface in any column (II). Results from a real-space renormalisation-group calculation (III) and Monte Carlo simulations (IV) are also displayed. The arrows denote the positions of the tricritical point and of the high-temperature series expansion result for the transition temperature of the spin-1 Ising model.

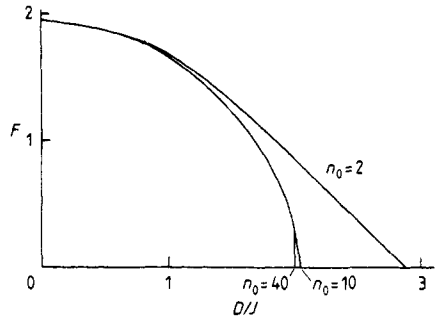


Figure 12. Dependence of the interface free energy calculated using the modified MHZ approximation on the crystal field for a value of the temperature for which the transition is first order. The dependence of the results on n_0 , the maximum allowed width of the interface within each column, is displayed. $k_B T/J = 0.5$ and $K/J = 0$.

as it passes through zero, it quickly sharpens as n_0 is increased, mirroring the free energy discontinuity expected at the transition.

(ii) $K/J \neq 0$

For $K/J \neq 0$ the model displays a rich phase diagram containing critical lines, triple lines, critical end-points and a Potts point (Berker and Wortis 1976). We studied in

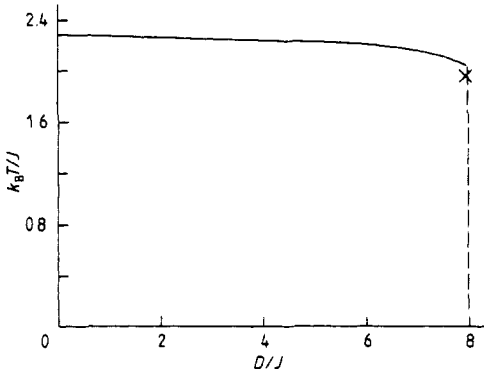


Figure 13. Phase diagram of the BEG model for $K/J = 3$ calculated using the modified MHZ approximation. The exact result for the three-state Potts transition is shown by a cross.

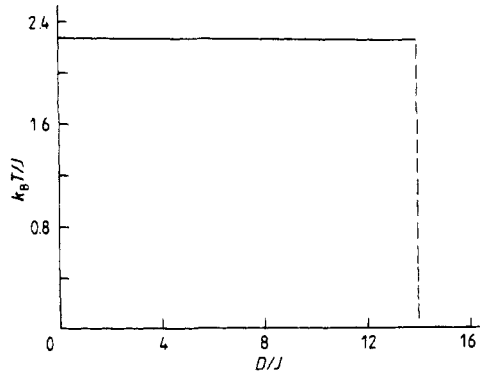


Figure 14. Phase diagram of the BEG model for $K/J = 6$ calculated using the modified MHZ approximation. The line of two-phase coexistence which is expected to extend beyond the end of the triple line is not seen within this approximation.

detail two cross-sections of the phase diagram, $K/J = 3$ and $K/J = 6$. The first case, $K/J = 3$, is of interest because the critical and triple lines join at a Potts tricritical point. The phase diagram predicted by the MHZ approximation is shown in figure 13 together with the exact result for the transition temperature of the three-state Potts model. The critical boundary is almost independent of the value of n_0 except near the Potts point. This indicates that the appearance of the interface is unfavourable as expected for large values of K . To obtain an accurate estimate for the position of the triple line results for $n_0 = \infty$, obtained by extrapolation, were used together with the exact zero-temperature value which can be extracted analytically from the approximation.

For $K/J = 6$ we expect that the tricritical point has been replaced by a critical end-point (Berket and Wortis 1976) and that a line of two-phase coexistence extends beyond the end of the triple line. As usual, it is not possible to identify two-phase coexistence within the MHZ approximation. However, as shown in figure 14, the critical and triple lines are readily identified.

4. Summary

In this paper we have presented the results of an investigation of the interface properties of the two-dimensional BEG model.

In particular, for the special case of the BC model, we have found, using MC techniques, that the net adsorption of the non-boundary state at the interface, W_0 , depends on the class of the bulk phase transition. W_0 remains finite on approach to the critical line, diverges logarithmically or with an exponent $\omega < 0.15$ at the tricritical point and diverges with an exponent $\omega \approx \frac{1}{3}$ in the vicinity of the first-order line.

Thus the critical behaviour of W_0 is *not* governed by the bulk correlation length. This is also the case for the interfacial wetting transition of the chiral Potts model (Huse and Fisher 1982) but not for the three-state Potts model.

Using a modified version of the Müller-Hartmann and Zittartz approximation we studied various sections of the phase diagram of the BEG model. The method provides a simple and rather precise way of estimating critical transition temperatures and lines of three-phase coexistence.

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